An Optimized Hybrid Model for Price Deviation Evaluation based on Neural Network

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Abstract

As a strong displaying approach, neural networks have been broadly utilized in finance and financial forecasting. Scientific and fair tradingtype open-end index fund (ETF) option pricing is favorable to maximizing the risk hedging function of the ETF, but it is also a sophisticated modelling procedure that necessitates a thorough understanding of market laws and economic importance. Based on China SSE 50 ETF, Harvest CSI 300 ETF, and high-frequency option data of 300ETF from Berry, this paper proposes another hybrid modelling method that consolidates the settled long-short-term memory neural network model (NLSTM) with the Heston model to acknowledge dynamic rectification of ETF choice price deviations. The research results show that the volatility characteristics of different types of ETF option prices are significantly different. Whether it is based on the BS pricing model or the Heston pricing model, it is difficult to accurately describe the complex changes in ETF option prices. By combining the NLSTM neural network model with the Heston model, it can effectively capture the dynamic changes of different types of ETF options, thereby improving the accuracy of ETF option pricing.

Keywords: information systems; queuing theory; revenue optimization; social networks

Introduction

Options are an important type of financial derivative products, and their pricing has been a hot issue that has attracted much attention for a long time. Since the launch of China's exchange-oriented open-end index fund (Exchanged Fund, ETF) options, ETF options products have played an important role in enriching trading strategies, enhancing the liquidity of the underlying, and ensuring the risk management of the financial market system (Jabeen et al., 2021). ETF options are a type of options with rapid development, active trading, and expanding influence. On the one hand, ETF options are Investors provide the opportunity to benefit from the return on basic assets at a lower cost Wen et al. (2021). On the other hand, by hedging the price fluctuations of the

underlying assets, investors can manage and control risks more flexibly(Liang et al., 2020). Therefore, the establishment of an efficient and accurate ETF option pricing model is an important research issue in the financial field.

The Black-Schools (BS) pricing model constructed by Black and Scholes Black and Scholes (1973) is currently the most widely used option pricing model. Limited by these assumptions, the BS model has also encountered some difficulties in practice, for example, the long memory and self-similarity of the price fluctuations of the underlying asset (Wingert et al.₉ 2020), the discontinuous

jump changes in the price of the underlying asset (Bormetti et al., 2020), and the implicit option Including volatility showing the phenomenon of "volatility smiling" (Branger et al. 2018) and the time-varying nature of interest rates (Bitto and Schnatter, 2018). The characteristics of production price movement (Golbabai and Nikan, 2019; Araneda, 2020; Ghasemalipour and Vajargah, 2019) of some scholars believe that interest rates will change over time, so they relax the assumption of constant interest rates in the BS model, and propose random interest rate models, such as Vasick random interest rate model (Homing and Chenfeng, 2010), Merton random interest rate Model (Merton, 1976), Hull-White random interest rate Model (Soleymani and Saray, 2019) and so on.

At present, the risk analysis and measurement research of the option market under the high-frequency environment has received extensive attention, and a series of breakthroughs have been made in the pricing research combining Reference Fan et al. (2018) used the Heston model to estimate the objective and risk-neutral density from the historical high-frequency data of the Dow Jones Index and its options, and then conducted an empirical study on the random discount factor. The implied volatility is extracted from the high-frequency data of the clock, and the change law of the price sensitivity of derivatives over time is discussed. Reference Muravyev and Ni (2020) explored the application of the BSM model in one-minute high-frequency S&P500 index options. In addition, scholars have also used Heston and other pricing models to develop a series of expansions and applications, such as

(Remer and Mahnke, 2004; Daniel et al., 2005; Noh and Kim, 2006) for stock index futures carried out relevant research with index constituent stocks.

Machine learning methods are just such a kind of theoretical methods that rely less on professional domain knowledge and do not need to consider data distribution assumptions and model assumptions. At present, many scholars have combined different types of machine learning methods to study option pricing problems, including support vector machines (Wang, 2011), Monte Carlo simulation (Leitao et al., 2017) and neural network models (Yao et al., 2000), etc. The above models are also EF option pricing the research provides ideas and references. At present, scholars have tried to use deep neural network models to study option pricing issues (Tong et al., 2020; Zhang et al., 2020).

Therefore, the integration of traditional option pricing models and neural networks has become a useful research perspective (Andreou et al., 2008; Liu et al., 2019; Huh, 2019). The robustness of the model also further increases the complexity of the hybrid model. In addition, most of the current option pricing research based on deep neural network models is based on direct modeling and forecasting of option prices, and less consideration is given to the research perspective of hybrid modeling of deep neural network models and traditional pricing models. Therefore, how to realize the organic combination of deep neural network model and option pricing model is a longterm prospect and research issue worthy of in-depth exploration in the field of rights pricing.

However, the neural network model also has shortcomings. For example, the neural network-based pricing model has insufficient adaptability to the financial sequence of intermittent transactions, and the modeling process lacks interpretability. From this, the combination of deep neural network theory and classic option pricing models for option pricing research can effectively overcome the respective problems of classic option pricing models and deep neural network models, such as nonlinear simulation. It combines it with the Heston pricing model to effectively reduce option pricing deviation and improve the risk prevention and control ability of decision makers. Then for the trading entities and financial institutions and the enterprise controls the risk and locks the cost to provide decision-making reference.

Literature Review

LSTM

The expression "LSTM" organizations, or "Long Short Term Memory" organizations, alludes to RNNs that are equipped for learning long term conditions. They were first presented by Hochreiter and Schmidhuber (1997), and numerous others in various areas cleaned and advocated them. In light of how well they work on different issues, they are right now habitually utilized in arrangement examination, discourse acknowledgment, and normal language handling. A hybrid LSTM and various GARCH model was utilized by Kim et al. to conjecture the unpredictability of the stock cost file. Liu et al. determined the breeze speed in Reference utilizing a hybrid model that consolidated the empirical wavelet transform, Elman neural network, and LSTM (Liu 2018 et al.). Long short term memory networks are explicitly intended to keep away from the drawn out reliance issue. They don't struggle with getting new material; as a matter of fact, it's practically similar to it easily falls into place for them to hold it for quite a while.

Price Prediction techniques

Computational intelligence techniques have generated a great deal of interest because of their outstanding capacity to handle challenging and nonlinear problems. With regards to anticipating power price, customary AI techniques like the extreme learning machine, support vector machine, and artificial neural network (ANN) have incredibly extraordinary results. The creator of consolidated a grouping strategy with an artificial neural network (ANN) half breed model for day-ahead cost guaging. Shrivastava et al. utilized information from the Ontario, New York, and Italian energy markets related to ELM and the wavelet way to deal with further develop determining unwavering quality and exactness. A model for projecting mid-term energy market clearing prices based on multiple support vector machines (SVM) is proposed by Yan et al. The proposed model is validated using PJM

interconnection data. Due to its outstanding performance in language modelling (Sundermeyer M et al.), speech recognition (Graves A, Jaitly N, Mohamed A), and natural language inference, deep learning has drawn a lot of attention as artificial intelligence has advanced swiftly (Chen Q, et al.).

Recurrent neural networks (RNNs) differ from traditional feed forward networks in that their neural connections can go either way, allowing neurons to pass information to the same layer or to a previous one. They are an efficient deep learning technique for processing sequential data, such as sound (Parascandolo G et al.) and stock market (Chen K et al.). However, the presence of "long-term dependencies" made it challenging for the conventional RNNs to analyse time series data.

ETF Option Pricing Theory Model

Heston Option Pricing Model

The BS model has become one of the exemplary hypothetical models in the act of choice pricing because of its reasonable rationale and clear process. The foundation of the BS model is primarily founded on the accompanying suspicions: the price change of the fundamental resource submits to mathematical Brownian movement. During the legitimacy time of the choice, the risk-free interest rate and unpredictability are consistent. There is no risk-free arbitrage an open door on the lookout. The BS model establishes the groundwork for the successful estimating of monetary subordinates like choices. However, with the deepening of option pricing theory and practice, the deficiencies of the large deviations between the assumptions of the BS model and many laws of the actual financial market have gradually been exposed (Barsotti and Sanfelici, 2016, Arabyat and AlZubi, 2022).

The Heston model relaxes the BS model's volatility constant assumption by taking into account that the price volatility of the underlying asset can fluctuate at will and combines the BS model with the CIR interest rate model (Ahlip and Rutkowski, 2013). The differential equation below determines the price of the underlying asset in the Heston model:
$$\begin{split} dS(t) &= \mu S(t) dt + \sqrt{V(t)} S(t) dW^{(1)}(t) dS(t) = \mu S(t) dt + \sqrt{V(t)} S(t) dW^{(1)}(t)_{(1)} \\ dV(t) &= \kappa (\theta - V(t)) dt + \sigma \sqrt{V(t)} dW^{(2)}(t) dV(t) = \kappa (\theta - V(t)) dt + \sigma \sqrt{V(t)} dW^{(2)}(t)_{(2)} \\ dW^{(1)}(t) dW^{(2)}(t) &= \rho dt \ dW^{(1)}(t) dW^{(2)}(t) = \rho dt \ _{(3)} \end{split}$$

Among them, S(t)S(t) reflects the underlying asset 's pricing at that time t t, V(t)V(t) represents the stochastic volatility of the price of the underlying asset at time t t, $\mu \mu$ represents the average return rate of the underlying asset, $\theta \theta$ shows the long-term average value of V(t)V(t), and $\kappa \kappa$ represents the recovery rate, $\sigma \sigma$ represents the volatility, $W^{(1)}(t)W^{(1)}(t)$ and $W^{(2)}(t)W^{(2)}(t)$ represent Brownian motion, and $\rho \rho$ shows the correlation coefficient of $W^{(1)}(t)W^{(1)}(t)$ and $W^{(2)}(t)W^{(2)}(t)$.

According to Eqs.(1)-(3), the partial differential equation of the option price C(S(t), V(t), t)C(S(t), V(t), t) with respect to the Heston model can be obtained, namely:

$$\frac{\partial C}{\partial t} + \frac{1}{2}VS^{2}\frac{\partial^{2}C}{\partial S^{2}} + \rho\sigma VS\frac{\partial^{2}C}{\partial S\partial V} + \frac{1}{2}\sigma^{2}V\frac{\partial^{2}C}{\partial V^{2}} + rS\frac{\partial C}{\partial S} + \left[\kappa(\theta - V(t)) - \lambda(S(t), V(t), t)\right]\frac{\partial C}{\partial V} - rC = \mathbf{0}$$

$$\frac{\partial C}{\partial t} + \frac{1}{2}VS^{2}\frac{\partial^{2}C}{\partial S^{2}} + \rho\sigma VS\frac{\partial^{2}C}{\partial S\partial V} + \frac{1}{2}\sigma^{2}V\frac{\partial^{2}C}{\partial V^{2}} + rS\frac{\partial C}{\partial S} + \left[\kappa(\theta - V(t)) - \lambda(S(t), V(t), t)\right]\frac{\partial C}{\partial V} - rC = \mathbf{0}$$
(4)

Among them, r r is the risk-free interest rate, and $\lambda(S(t), V(t), t)\lambda(S(t), V(t), t)$ is the market price of volatility risk.

As per Eqs.(1)- (4), the explicit solution equation of European call choices under the Heston model can be gotten by the technique for inductive proof, in particular:

 $C(S(t), V(t), t) = S(t)P_1 - Ke^{-r(T-t)}P_2C(S(t), V(t), t) = S(t)P_1 - Ke^{-r(T-t)}P_2(5)$

Among them, K K is the exercise price of a European call option, T T is the expiration time of the option, and $P_1 P_1$ and $P_2 P_2$ are two probability distribution functions:

$$P_{J} = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{+\infty} \mathbb{R}e\left[\frac{\exp\exp\left(-i\varphi \ln\ln(K)f_{j}(x,V(t),T-t,\varphi)\right)}{i\varphi}\right] d\varphi$$

$$P_{J} = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{+\infty} \mathbb{R}e\left[\frac{\exp\exp\left(-i\varphi \ln\ln(K)f_{j}(x,V(t),T-t,\varphi)\right)}{i\varphi}\right] d\varphi$$
(6)

Assuming that f_1f_1 and f_2f_2 are the characteristic functions of P_1P_1 and P_2P_2 , then f_1f_1 and f_2f_2 satisfy the following functional forms:

 $\begin{aligned} f_i(x, V(t), t, \varphi) &= \exp \exp \{i\varphi x + C_i(T - t, \varphi) + D_i(T - t, \varphi)V(t)\} \\ f_j(x, V(t), t, \varphi) &= \exp \exp \{i\varphi x + C_j(T - t, \varphi) + D_j(T - t, \varphi)V(t)\}_{(7)} \end{aligned}$

In the above formula,

$$\begin{aligned} x &= \ln\ln(S(t))x = \ln\ln(S(t))_{(8)} \\ C_j(t,\varphi) &= ri\varphi(T-t) + \frac{a}{\sigma^2}(b_j - \rho\sigma\varphi i + d_j)(T-t) - 2\ln\ln\left(\frac{1 - g_j\exp\exp(d_j(T-t))}{1 - g_j}\right) \\ C_j(t,\varphi) &= ri\varphi(T-t) + \frac{a}{\sigma^2}(b_j - \rho\sigma\varphi i + d_j)(T-t) - 2\ln\ln\left(\frac{1 - g_j\exp\exp(d_j(T-t))}{1 - g_j}\right)_{(9)} \end{aligned}$$

$$D_{j}(T-t,\varphi) = \frac{b_{j} - \rho\sigma\varphi i + d_{j}}{\sigma^{2}} \left(\frac{1 - \exp\exp\left(d_{j}(T-t)\right)}{1 - g_{j}\exp\exp\left(d_{j}(T-t)\right)} \right)$$

$$D_{j}(T-t,\varphi) = \frac{b_{j} - \rho\sigma\varphi i + d_{j}}{\sigma^{2}} \left(\frac{1 - \exp\exp\left(d_{j}(T-t)\right)}{1 - g_{j}\exp\exp\left(d_{j}(T-t)\right)} \right)_{(10)}$$

$$g_{j} = \frac{b_{j} - \rho\sigma\varphi i + d_{j}}{b_{j} - \rho\sigma\varphi i - d_{j}} g_{j} = \frac{b_{j} - \rho\sigma\varphi i + d_{j}}{b_{j} - \rho\sigma\varphi i - d_{j}} (11)$$

$$d_{j} = \sqrt{\left(\rho\sigma\varphi i - b_{j}\right)^{2} - \sigma^{2}\left(2u_{j}\varphi i - \varphi^{2}\right)} d_{j} = \sqrt{\left(\rho\sigma\varphi i - b_{j}\right)^{2} - \sigma^{2}\left(2u_{j}\varphi i - \varphi^{2}\right)} (12)$$

where, j = 1, 2, j = 1, 2, $\alpha_1 = \frac{1}{2}\alpha_1 = \frac{1}{2}$, $\alpha_2 = -\frac{1}{2}\alpha_2 = -\frac{1}{2}$, $\kappa^* = \kappa + \lambda \kappa^* = \kappa + \lambda$, $\theta^* = \frac{\kappa\theta}{\kappa} + \lambda$, $\theta^* = \kappa + \lambda - \rho\sigma$, $b_2 = \kappa + \lambda$, $b_2 = \kappa + \lambda$, $\alpha = \kappa^*\theta^*\alpha = \kappa^*\theta^*$, $\lambda \lambda$ is the

market price of volatility risk, Re(x)Re(x) shows the real part of $x \ x$, and $i \ i$ is the imaginary unit. According to the characteristic functions $f_1 f_1 \text{ and } f_2 f_2$, perform the inverse Fourier transform to obtain the probability distribution functions $P_1 P_1$ and $P_2 P_2$. Here, the parameters $\kappa \kappa$, $\theta \theta$, $\sigma \sigma$, $\rho \rho$, V(t)V(t), $\lambda \lambda$ need to be determined to calculate the pri ce of European call option. However, the volatility risk premium $\lambda \lambda$ is not a tradable commodity in the market, and it is difficult to observe its price, so it is difficult to estimate $\lambda \lambda$.

NLSTM Model

LSTM neural network is a further development based o n recurrent neural network , which is reasonable for handling time series with long term condit ions. In the LSTM NN model, the update formula and gating system of the unit state are communicated by the accompanying equation:

$$i_{t} = \sigma_{i}(W_{xi}x_{t} + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_{i})i_{t} = \sigma_{i}(W_{xi}x_{t} + W_{hi}h_{t-1} + W_{ci}c_{t-1} + b_{i})_{(13)}$$

$$f_{t} = \sigma_{f}(W_{xf}x_{t} + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_{f})f_{t} = \sigma_{f}(W_{xf}x_{t} + W_{hf}h_{t-1} + W_{cf}c_{t-1} + b_{f})_{(14)}$$

$$c_{t} = f_{t \odot c_{t-1}} + i_{t \odot \sigma_{c}}(W_{xc}x_{t} + W_{hc}h_{t-1} + b_{c})c_{t} = f_{t \odot c_{t-1}} + i_{t \odot \sigma_{c}}(W_{xc}x_{t} + W_{hc}h_{t-1} + b_{c})_{(15)}$$

$$o_{t} = \sigma_{0}(W_{xo}x_{t} + W_{ho}h_{t-1} + b_{o})o_{t} = \sigma_{0}(W_{xo}x_{t} + W_{ho}h_{t-1} + b_{o})_{(16)}$$

$$h_{t} = o_{t \odot \sigma_{h}(c_{t})}h_{t} = o_{t \odot \sigma_{h}(c_{t})(17)}$$

Different from the traditional LSTM neural n etwork model, the NLSTM replaces the update method of the unit state, and uses a new calculation method $c_t = m_t (f_{t \odot c_{t-1}}, i_{t \odot g_t}) c_t = m_t (f_{t \odot c_{t-1}}, i_{t \odot g_t})$ to replace the addition operation of $c_t c_t$ in the traditional LSTM, where $m_t m_t$ represents the memory state inside the neural network unit at the time t t, and are recursively calculated $c_t c_t$ and $m_t m_t$.

This constitutes the nested structure of the NL -STM neural network [39 - 40]. The input and hidden state of the internal nested structure (memory function) in the NLSTM neural network can be represented by the following functions:

$$\begin{split} \widetilde{\mathbf{h}}_{t-1} &= f_{t \otimes c_{t-1}} \widetilde{\mathbf{h}}_{t-1} = f_{t \otimes c_{t-1}}(18) \\ \widetilde{\mathbf{x}}_{t} &= i_{t \otimes \sigma_{c}} (\mathbf{x}_{t} W_{\mathbf{x}c} + \mathbf{h}_{t-1} W_{\mathbf{h}c} + b_{c}) \widetilde{\mathbf{x}}_{t} = i_{t \otimes \sigma_{c}} (\mathbf{x}_{t} W_{\mathbf{x}c} + \mathbf{h}_{t-1} W_{\mathbf{h}c} + b_{c}) (19) \\ c_{t} &= \widetilde{\mathbf{h}}_{t-1} + \widetilde{\mathbf{x}}_{t} c_{t} = \widetilde{\mathbf{h}}_{t-1} + \widetilde{\mathbf{x}}_{t} (20) \\ \widetilde{\mathbf{t}}_{t} &= \widetilde{\sigma}_{i} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}i} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}i} + \widetilde{b}_{i}) \widetilde{\mathbf{t}}_{t} = \widetilde{\sigma}_{i} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}i} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}i} + \widetilde{b}_{i}) (21) \\ \widetilde{f}_{t} &= \widetilde{\sigma}_{f} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}f} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}f} + \widetilde{b}_{f}) \widetilde{f}_{t} = \widetilde{\sigma}_{f} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}f} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}f} + \widetilde{b}_{f}) (22) \\ \widetilde{c}_{t} &= \widetilde{f}_{t \otimes \widetilde{c}_{t-1}} + \widetilde{t}_{t \otimes \widetilde{\sigma}_{c}} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}c} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}c} + \widetilde{b}_{c}) \widetilde{c}_{t} = \widetilde{f}_{t \otimes \widetilde{c}_{t-1}} + \widetilde{t}_{t \otimes \widetilde{\sigma}_{c}} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}c} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}c} + \widetilde{b}_{c}) (23) \\ \widetilde{o}_{t} &= \widetilde{\sigma}_{\mathbf{0}} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}o} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}o} + \widetilde{b}_{o}) \widetilde{o}_{t} = \widetilde{\sigma}_{\mathbf{0}} (\widetilde{\mathbf{x}}_{t} \widetilde{W}_{\mathbf{x}o} + \widetilde{\mathbf{h}}_{t-1} \widetilde{W}_{\mathbf{h}o} + \widetilde{b}_{c}) (24) \\ \widetilde{\mathbf{h}}_{t} &= \widetilde{\sigma}_{t \otimes \widetilde{\sigma}_{\mathbf{h}}(\widetilde{c}_{t})} \widetilde{\mathbf{h}}_{t} = \widetilde{\sigma}_{t \otimes \widetilde{\sigma}_{\mathbf{h}}(\widetilde{c}_{t}) (25) \end{split}$$

Modeling Framework

According to the viewpoint of monetary time series instability investigation, the center of price estimating is to get a handle on the irregular leap, dynamic and different qualities of the grouping (Liu and Pan, 2003), and build a scientific theoretical model on this basis to analyze the volatility trend of option prices. However, the classic BS model, Heston model and other models have encountered many bottlenecks in practical applications, for example: the distribution of option prices and other characteristics do not match the assumptions of the classic model. There are often certain differences between different types of option targets. The adaptability and robustness of pricing models built for specific sequence characteristics and distribution characteristics are often insufficient (Peng and Hu, 2020; Cook and Johannsdottir, 2021), traditional option pricing models are prone to deviations. The NLSTM neural network model is introduced for hybrid construction. The proposed hybrid pricing modeling framework is shown in Fig. 1.



Figure 1: Proposed framework

The ETF option hybrid pricing modeling framework based on NLSTM and Heston has the following steps:

Step 1: Heston model parameter estimation. For parameter estimation, the least squared error between the Heston model estimated price and the real market price is required, combined with trust region reflection (TRR) algorithm searches and solves the objective function, and then estimates the five parameters $\kappa \kappa$, $\theta \theta$, $\sigma \sigma$, $\rho \rho$ and V(t)V(t)

Step 2: The grouping of deviations is extricated. The contrast between the estimated price of the Heston model and the real option price is determined, and the estimating deviation arrangement is normalized, involving the genuine ETF option price as the benchmark.

Step 3: Developing a NLSTM NN model. Partition the example sets into preparing and test sets, select the organization structure for the NLSTM NN, and train the NLSTM neural network utilizing the example set made utilizing the normalized cost deviation arrangement and Heston's unique information ideas.

Step 4: The pricing error has been corrected. Analyze the extension of the deviation sequence using the trained NLSTM neural network model, integrate the NLSTM neural network model's extension estimation result with the Heston model's projected price, correct the ETF option pricing results, and finish the final ETF option pricing.

Empirical Analyses

Data Source and Sample Selection

This article analyzes the one-minute call option trading data of China SSE 50ETF, Harvest CSI 300 ETF, and Huatai Bai Rui CSI 300 ETF as examples. From March 06 to July 17, 2020, the time range of Harvest CSI 300 ETF options is from February 03, 2020 to July 2020.On March 17, the data comes from the WIND database. The strike prices of the options are Huaxia SSE 50ETF call option (2.8000 yuan), Harvest CSI 300ETF call option (3.8000 yuan) and Huataibai Rui CSI 300ETF call option (3.8000 yuan). In addition, the expiration dates of the options in the sample are all on September 23, 2020. For the risk-free interest rates of other remaining maturities, use the adjacent maturity interest rate is calculated by linear interpolation.

After preprocessing the original data, after removing the samples with null option closing prices and null closing prices of the underlying assets, the basic points of the option samples is shown in Table. 1.

This article categorizes option samples according to the difference in the remaining maturity of options, and on this basis, the samples of the remaining period of the same kind, the data of the first two days of the sample is selected for parameter estimation, and the remaining samples are used for model verification. See Tab. 2 for the specific division.

Table 1. Dasic information table of option samples				
Option data	The amount of sample data	Moneyness	Underlying asset price	Expire date
China SSE 50 ETF	7183	[0.98, 1.27]	[2.75, 2.53]	[69, 141]
Harvest CSI 300 ETF	2116	[0.93, 1.32]	[3.55, 5]	[69, 234]
Huatai Bai Rui Shanghai and Shenzhen 300 ETF	5247	[1, 1.3]	[3.8, 4.92]	[69, 141]

Table 1: Basic information table of option samples

Option data	Parameter Estimation	Training and test data set values	
		Training set	Test set
	05.06-05.07	05.08-05.26	
China SSE 50 ETF	05.27-05.28	05.29-06.28	07.07-07.17
	06.29-06.30	07.01-07.06	
	02.03-02.04	02.05-02.26	
Harvest CSI 300ETF	02.27-02.28	02.29-03.29	
	03.30-03.31	04.01-04.26	
	04.27-04.28	04.29-05.26	07.07-07.17
	05.27-05.28	05.29-06.28	
	06.29-06.30	07.01-07.06	
Huatai Bai Rui Shanghai and	05.06-05.07	05.08-05.26	
Shenzhen 300ETF	05.27-05.28	05.29-06.28	07.07-07.17
	06.29-06.30	07.01-07.06	

Table 2: Division of experimental data

Experimental Parameter Settings

The modified hybrid pricing model based on NLSTM builds the NLSTM model under the Keras framework, hereinafter referred to as the NLSTM model. This model contains a layer of LSTM and a layer of nested LSTM, and the corresponding numbers of neurons are 64 and 32, respectively. In order to prevent over-fitting in model training, the Dropout method is used in the last layer to randomly remove some hidden neurons. The hybrid pricing model based on LSTM correction is also constructed under the Keras framework. The LSTM model contains three

layers of LSTM, and the no of neurons in every layer is 64, 32, and 32, respectively. The other settings of the model are consistent with the NLSTM model.

The training must decide the size of each batch of data fed into the LSTM or NLSTM model (Batch-size), as well as the number of training sample sets, in addition to selecting the model parameters (Epoch). The selection of Batch-size includes four values of 16, 32, 64 and 128, and then the appropriate Epoch is selected through the Loss value curve. From the figure 2, we can see that the curves of the three sets of data tend to be stable when Epoch is 20, 20, and 30, so Epoch selects 20, 20, and 30. The remaining comparison model parameters are shown in Tab. 3.





(b) Options for Harvest CSI 300 ETF



(c) Optional Huatai Berry CSI 300 ETFs

Model	Parameter name	Huatai Bai Rui Shanghai and Shenzhen 300ETF	Harvest CSI 300ETF	China SSE 50ETF
BS	σ	0.1573	0.2491	0.1682
Heston	κ	2.9643	2.8025	0.0296
	θ	0.0669	0.1062	5
	σ	0.3765	0.6794	0.7392
	ρ	1.0129	0.9557	0
	V(0)	0.0006	0.0026	0
LSTM	Bacthsize	32	64	16
	Epoch	20	20	20
BS-LSTM	Bacthsize	32	128	64
	Epoch	20	20	10
Heston-LSTM	Bacthsize	64	128	32
	Epoch	30	20	20
BS-NLSTM	Bacthsize	32	64	64
	Epoch	20	20	20
Heston-NLSTM	Bacthsize	64	128	64
	Epoch	30	20	20

Figure 2: Comparison of loss change of training set of Heston-NLSTM hybrid pricing model Table 3: Parameter settings of the corresponding data sets for different models

In addition, the input attributes of the proposed hybrid pricing model and comparison model also need to be determined.

This experiment chooses Moneyness greater than 1.05 (ITM, real option), and the actual expiration days/366 days greater than 0.2 (calculated based on 366 days in 2020, that is, the remaining Experiment with option data with duration greater than 74 days. The corresponding calculation formulas are as follows:

$$MAPE = \sum_{i=1}^{n} \Box \left| \frac{obs_t - est_t}{obs_t} \right| \times \frac{100}{n} MAPE = \sum_{i=1}^{n} \Box \left| \frac{obs_t - est_t}{obs_t} \right| \times \frac{100}{n} (27)$$
$$MSE = \frac{1}{n} \sum_{i=1}^{n} \Box (obs_t - est_t)^2 MSE = \frac{1}{n} \sum_{i=1}^{n} \Box (obs_t - est_t)^2 (28)$$

Among them, *obs obs* is the true value of the sample, and *est est* is the estimated value of the sample.

Analysis of the Results of China SSE 50E Option Data

Fig. 3 shows the correlation of the fitting aftereffects of various models applied to the China SSE 50ETF choice informational index. The figure shows that each model has a stronger fitting impact at the beginning of the model preparation process. With the increase of the latter option price fluctuation range, the fitting effect of traditional models such as Heston begins to deteriorate. Fig. 4 compares the option price estimates of different models. The BS model and the Heston model have lower pricing accuracy than the deep neural network model, and there is an obvious tendency to underestimate the price. The related pricing model based on neural network shows that the early estimation results are lower than the real data, and the later estimation results are generally higher than the real data and gradually approach the real value. Among them, the proposed model has a relatively better fit for option prices.



Figure 3: Comparison of training results and real values of different models applied to China SSE 50ETF option data



Figure 4: Comparison of estimated value and real value of different models applied to China SSE 50ETF option data

Tab. 4 demonstrates that the proposed model's MAPE value is the lowest when compared to the BS model, Heston model, LSTM model, BS-LSTM model, Heston-LSTM model, and the BS-NLSTM model, and is 2.560%, 2.519%, 0.721%, 0.035%, 0.071%, and 0.068% lower, demonstrating the suggested model's pricing accuracy.

Table 4:Comparison of price outcomes from several models used to the data from the China SSE 50E option (calculate the average value of 10 times)

Model	MSE (%)	MAPE (%)
BS	0.165	4.801
Heston	0.163	4.760
LSTM	0.077	2.962
BS-LSTM	0.048	2.276
Heston-LSTM	0.050	2.312
BS-NLSTM	0.052	2.309
Heston-NLSTM	0.051	2.241

Analysis of Harvest CSI 300 ETF Option Data Results

Figure 5 compares the outcomes of fitting various models to the data set for the Harvest CSI 300 ETF option. The chosen Castrol CSI 300 ETF option price varies significantly due to significant data variations, as seen in the figure. The standard pricing model has a poor fitting impact, while the BS model has a propensity to overestimate.



The price estimation of various models used on the Harvest Shanghai and Shenzhen 300 ETF option data set is shown in Fig. 6. In the later stage of option price estimation, the degree of fit between the BS-NLSTM and the proposed model is better.





Combining the results from Fig. 6 and Tab. 5, it is easy to see that the hybrid pricing model suggested in this article does not have the highest estimation accuracy of all models for the Harvest CSI 300 ETF option data (MAPE is 1.942%), but is only comparable with the BS model, with the highest accuracy dropped by 0.062%, while the robustness of the model in this paper is the highest (MSE is 0.073%), which also demonstrates its effectiveness.

Table 5: Comparison of pricing results of different m	odels
applied to Harvest CSI 300E option data (calculat	e the
average value of 10 times)	

Model	MSE (%)	MAPE (%)
BS	0.080	1.880
Heston	0.091	2.047
LSTM	0.098	2.264
BS-LSTM	0.084	2.089
Heston-LSTM	0.092	2.299
BS-NLSTM	0.179	2.232
Heston-NLSTM	0.073	1.942

Analysis of the Results of Huatai Bairui Shanghai and Shenzhen 300E Option Data

Fig. 7 shows the examination of the attack of various models applied to the Huatai Bairui Shanghai and Shenzhen 300 ETF choice information. Fig. 8 shows the examination between the assessed esteem and the genuine

worth of option price in various models. It very well may be seen that, the exactness of the LSTM model in the beginning phase of assessment is higher, and each pricing model tends to underrate, however the assessment precision of the LSTM model in the later phase of assessment drops essentially.







Figure 8: Comparison of estimated value and real value of different models applied to Huatai Bairui Shanghai and Shenzhen 300ETF option data

It tends to be seen from Tab. 6 that, the proposed hybrid pricing model has the most exorbitant price assessment exactness (MAPE is 1.854%, MSE is 0.058%). Contrasted and the LSTM model, the Heston-LSTM model and the BS-NLSTM model, they are lower. The outcomes were 0.154%, 0.141%, 0.604%, 0.143%, 0.004% and 0.159%. The hybrid pricing model suggested in this article maintains the advantages of the traditional option pricing model's thorough justification, straightforward construction, and low computational complexity while taking into account the ability to accurately depict nonlinear patterns, according to the aforementioned results.

Table 6: Comparison of pricing results of different modelsapplied to Huatai Bairui Shanghai and Shenzhen 300Eoption data (calculate the average value of 10 times)

Model	MSE (%)	MAPE (%)
BS	0.071	2.008
Heston	0.070	1.995
LSTM	0.097	2.458
BS-LSTM	0.067	1.997
Heston-LSTM	0.062	1.858
BS-NLSTM	0.065	2.013
Heston-NLSTM	0.058	1.854

Conclusion

The NLSTM model, which combines the deep neural network model and the conventional option pricing approach, is the foundation of the unique ETF option pricing methodology presented in this study. It uses highfrequency option data from the China SSE 50 ETF, Huatai Bai Rui CSI 300 ETF, and Harvest CSI 300 ETF. Based on this, it fits the option pricing deviation and modifies the option prices using the estimation findings of the extension of pricing deviations. It analyses and contrasts several pricing models as well as the precision of the pricing analysis of the models put out in this research. It is challenging to adjust to the complicated changing laws of ETF option prices since the experimental results demonstrate that the volatility characteristics of various types of ETF option prices fluctuate greatly. The hybrid pricing model suggested in this paper can more accurately reflect the dynamic fluctuations of various types of ETF options by including the NLSTM neural network model. It is therefore not difficult to conclude that in order to accomplish an accurate calculation of the ETF option price, thorough consideration of the actual price formation mechanism is required. However, it is frequently difficult for the LSTM model of the deep neural network to accurately account for the aforementioned mechanisms, and as a result, its interpretation is subpar. With the help of the feature capturing ability and nonlinear fitting ability of the NLSTM model, this paper combines it with the Heston pricing model to effectively reduce option pricing deviation and improve the risk prevention and control ability of decision makers.

The research in this article also has some shortcomings. Future research should focus on critical questions including how to build a hybrid pricing model that is more reliable and useful as well as how to price various types of options using the approach described in this article.

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